$\boxed{\text{Ch. 7 Review}}$

1.) $\int_1^2 \frac{(x+1)^2}{x}\,dx = \int_1^2 \frac{x^2+2x+1}{x}\,dx = \int_1^2 \left(x+2+\frac{1}{x}\right)dx$

$= \frac{1}{2}x^2 + 2x + \ln x\Big]_1^2 = \left(\frac{1}{2}(2)^2+2(2)+\ln(2)\right)-\left(\frac{1}{2}+2+0\right)$

$\cancel{2}+4+\ln(2)-\frac{1}{2}-\cancel{2}$

$= \boxed{\frac{7}{2}+\ln(2)}$

3.) $\int \frac{e^{\sin(x)}}{\sec(x)}\,dx = \int e^{\sin(x)}\cos(x)\,dx$ $\left(\begin{array}{l} u=\sin(x) \\ du=\cos(x)dx \end{array}\right)$

$\int e^u du = e^u + C = \boxed{e^{\sin(x)}+C}$

5.) $\int \frac{dt}{2t^2+3t+1} = \int \frac{1}{(2t+1)(t+1)}\,dt$

$\frac{1}{(2t+1)(t+1)} = \frac{A}{2t+1}+\frac{B}{t+1}$

$1 = A(t+1)+B(2t+1)$

$1 = At + A + 2Bt + B)$

$0 = A + 2B$

$-1\,(1 = A + B)$

$-1 = -A - B$

$0 = A + 2B$

$-1 = B$

$2 = A$

$\int\left(\frac{2}{2t+1}+\frac{-1}{t+1}\right)dt =$

$\int \frac{2}{2t+1}\,dt - \int \frac{1}{t+1}\,dt$ $\quad u_2 = t+1$, $du_2 = dt$

$u_1 = 2t+1$

$du_1 = 2dt$

$\int \frac{1}{u_1}du - \int \frac{1}{u_2}du$

$\ln|u_1| - \ln|u_2| + C$

$\boxed{\ln|2t+1| - \ln|t+1| + C}$

7.) $\int_0^{\pi/2} \sin^3\theta \cos^2\theta \, d\theta = \int_0^{\pi/2} \sin^2\theta \cos^2\theta \sin\theta \, d\theta =$

$\int_0^{\pi/2} (1-\cos^2\theta)\cos^2\theta \sin\theta \, d\theta \quad \begin{pmatrix} u = \cos\theta \\ du = -\sin\theta \, d\theta \end{pmatrix}$

$-\int_1^0 (1-u^2)(u^2)\,du = -\int_1^0 u^2 - u^4 \, du = \int_0^1 u^2 - u^4 \, du =$

$\frac{1}{3}u^3 - \frac{1}{5}u^5 \Big]_0^1 = \left(\frac{1}{3} - \frac{1}{5}\right) - 0 = \frac{5}{15} - \frac{3}{15} = \boxed{\frac{2}{15}}$

9.) $\int \frac{\sin(\ln t)}{t} \, dt \quad \begin{pmatrix} u = \ln t \\ du = \frac{1}{t} dt \end{pmatrix} \quad \int \sin u \, du =$

$-\cos u + C = \boxed{-\cos(\ln t) + C}$

11.) $\int_1^2 \frac{\sqrt{x^2-1}}{x} \, dx \quad$ Let $x = \sec\theta$, $dx = \sec\theta\tan\theta \quad \int_0^{\pi/3} \frac{\sqrt{\sec^2\theta - 1}}{\sec\theta} \sec\theta\tan\theta \, d\theta =$

$\int_0^{\pi/3} \frac{\sqrt{\tan^2\theta}}{\sec\theta} \sec\theta\tan\theta \, d\theta = \int_0^{\pi/3} \frac{\tan\theta}{\cancel{\sec\theta}} \cancel{\sec\theta}\tan\theta \, d\theta = \int_0^{\pi/3} \tan^2\theta \, d\theta =$

$\int_0^{\pi/3} (\sec^2\theta - 1)\,d\theta = \tan\theta - \theta \Big]_0^{\pi/3} \quad \tan\left(\frac{\pi}{3}\right) - \frac{\pi}{3} - 0 = \boxed{\sqrt{3} - \frac{\pi}{3}}$

13.) $\int e^{\sqrt[3]{x}}\,dx$ $\left(\begin{array}{c} w=\sqrt[3]{x} \\ w^3=x \\ 3w^2dw=dx \end{array}\right)$ $\int e^{w}\cdot 3w^2dw$ $\quad u=w^2 \quad dv=e^w dw$

$du=2wdw \quad v=e^w$

(use substitution) $3\int e^w w^2 dw$

(use integration by parts)

$=\int e^w\cdot w^2dw = e^w w^2-\int e^w 2wdw$ (use integration by parts again)

$u=w \quad dV=e^w dw$

$du=dw \quad V=e^w$ $\longrightarrow$ $\int e^w w^2 dw = w^2e^w-2\left(we^w-\int e^w dw\right)$

$= w^2e^w-2we^w+2e^w+C$

$3\int e^w w^2 dw = 3\left(w^2e^w-2we^w+2e^w\right)+C = 3e^w(w^2-2w+2)+C$

(back substitute) $\boxed{3e^{\sqrt[3]{x}}(x^{2/3}-2x^{1/3}+2)+C}$

15.) $\int \frac{x-1}{x^2+2x}\,dx$ $\quad \frac{x-1}{x(x+2)}=\frac{A}{x}+\frac{B}{x+2} \Rightarrow x-1=A(x+2)+Bx$

$x-1=Ax+2A+Bx$

$1=A+B$

$-1=2A$

$1/2=-A$

$3/2=B$

$-1/2=A$

$\int\left(\frac{-1/2}{x}+\frac{3/2}{x+2}\right)dx = -\frac{1}{2}\int\frac{1}{x}dx+\frac{3}{2}\int\frac{1}{x+2}dx$

$\boxed{-\frac{1}{2}\ln|x|+\frac{3}{2}\ln|x+2|+C}$

17.) $\int x\cosh(x)\,dx$ $\left(\begin{array}{cc} u=x & dv=\cosh(x)\,dx \\ du=dx & v=\sinh(x) \end{array}\right)$

$= x\sinh(x)-\int\sinh(x)\,dx$

$\boxed{= x\sinh(x)-\cosh(x)+C}$

19.) $\displaystyle\int \frac{x+1}{9x^2+6x+5}\,dx = \int \frac{x+1}{(9x^2+6x+1)+4}\,dx = \int \frac{x+1}{(3x+1)^2+4}\,dx$

(perfect sq. trinomial)

let $u = 3x+1 \rightarrow \frac{1}{3}(u-1) = x$

$du = 3dx$

$\frac{1}{3}du = dx$

$\displaystyle\int \frac{\left[\frac{1}{3}(u-1)\right]+1}{u^2+4}\,\frac{1}{3}du =$

$\displaystyle\frac{1}{3}\cdot\frac{1}{3}\int \frac{(u-1)+3}{u^2+4}\,du = \frac{1}{9}\int \frac{u+2}{u^2+4}\,du = \frac{1}{9}\int \frac{u}{u^2+4}\,du + \frac{1}{9}\int \frac{2}{u^2+4}\,du$

(let $w = u^2+4$

$dw = 2du$

$\frac{1}{2}dw = du$)

$\displaystyle\frac{1}{9}\int \frac{1}{w}\cdot\frac{1}{2}dw + \frac{2}{9}\int \frac{1}{u^2+2^2}\,du = \frac{1}{18}\int \frac{1}{w}\,dw + \frac{2}{9}\cdot\frac{1}{2}\tan^{-1}\left(\frac{1}{2}u\right) + C$

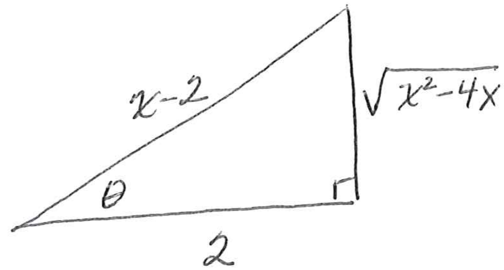
$= \frac{1}{18}\ln|w| + \frac{1}{9}\tan^{-1}\left(\frac{1}{2}u\right) + C = \frac{1}{18}\ln(u^2+4) + \frac{1}{9}\tan^{-1}\left(\frac{1}{2}u\right) + C$

$= \frac{1}{18}\ln\left((3x+1)^2+4\right) + \frac{1}{9}\tan^{-1}\left(\frac{1}{2}(3x+1)\right) + C$

$= \boxed{\frac{1}{18}\ln\left(9x^2+6x+5\right) + \frac{1}{9}\tan^{-1}\left(\frac{1}{2}(3x+1)\right) + C}$

21.) $\int \frac{dx}{\sqrt{x^2-4x}} = \int \frac{dx}{\sqrt{(x^2-4x+4)-4}} = \int \frac{dx}{\sqrt{(x-2)^2-2^2}}$ $\left(\begin{array}{l} x-2 = 2\sec\theta \\ dx = 2\sec\theta\tan\theta \, d\theta \end{array} \right)$

$$\int \frac{2\sec\theta\tan\theta\, d\theta}{\sqrt{4\sec^2\theta - 4}} = \int \frac{2\sec\theta\tan\theta\, d\theta}{\sqrt{4\tan^2\theta}} = \int \frac{\cancel{2}\sec\theta\,\cancel{\tan}\theta\, d\theta}{\cancel{2}\cancel{\tan}\theta}$$



$$\int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C_1$$

$$= \ln\left|\frac{x-2}{2} + \frac{\sqrt{x^2-4x}}{2}\right| + C_1$$

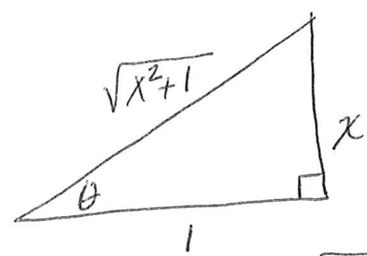
$$= \ln\left|\tfrac{1}{2}\left(x-2+\sqrt{x^2-4x}\right)\right| + C_1$$

$$= \ln\left(\tfrac{1}{2}\right) + \ln\left|x-2+\sqrt{x^2-4x}\right| + C_1$$

$$= -\ln(2) + \ln\left|x-2\sqrt{x^2-4x}\right| + C_1$$

23.) $\int \frac{dx}{x\sqrt{x^2+1}}$ $\left(\begin{array}{l} x = \tan\theta \\ dx = \sec^2\theta\, d\theta \end{array} \right)$ $\int \frac{\sec^2\theta\, d\theta}{\tan\theta\sqrt{\tan^2\theta+1}} = \int \frac{\sec^2\theta\, d\theta}{\tan\theta\sqrt{\sec^2\theta}}$

$$= \int \frac{\sec^2\theta\, d\theta}{\tan\theta\sec\theta} = \int \frac{\sec\theta}{\tan\theta}\, d\theta = \int \csc\theta\, d\theta = \ln|\csc\theta - \cot\theta| + C$$



$$\ln\left|\frac{\sqrt{x^2+1}}{x} - \frac{1}{x}\right| + C =$$

$\csc\theta = \frac{\sqrt{x^2+1}}{x}$

$\cot\theta = \frac{1}{x}$

$$\boxed{\ln\left|\frac{\sqrt{x^2+1}-1}{x}\right| + C}$$

25.) $\int \frac{3x^3 - x^2 + 6x - 4}{(x^2+1)(x^2+2)}\,dx = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2}$

$$3x^3 - x^2 + 6x - 4 = (Ax+B)(x^2+2) + (Cx+D)(x^2+1)$$

$$3x^3 - x^2 + 6x - 4 = Ax^3 + Bx^2 + 2Ax + 2B + Cx^3 + Dx^2 + Cx + D$$

$3 = A + C$

$-1 = B + D$

$6 = 2A + C$

$-4 = 2B + D$

$-6 = -\cancel{2}A - 2C$

$6 = \cancel{2}A + C$

$0 = -C$

$0 = C$

$3 = A$

$2 = -\cancel{2}B - 2D$

$-4 = \cancel{2}B + D$

$-2 = -D$

$2 = D$

$-1 = B + 2$

$-3 = B$

$$\int \frac{3x-3}{x^2+1} + \frac{2}{x^2+2}\,dx = 3\int \frac{x-1}{x^2+1}\,dx + 2\int \frac{1}{x^2+2}\,dx$$

$\left(\begin{array}{l} u = x^2+1 \\ du = 2x\,dx \\ \frac{1}{2}du = x\,dx \end{array} \right)$

$$= 3\int \frac{x}{x^2+1}\,dx - 3\int \frac{1}{x^2+1}\,dx + 2\int \frac{1}{x^2+2}\,dx$$

$$= \frac{3}{2}\int \frac{1}{u}\,du - 3\int \frac{1}{x^2+1}\,dx + 2\int \frac{1}{x^2+(\sqrt{2})^2}\,dx$$

$$= \frac{3}{2}\ln|u| - 3\tan^{-1}(x) + \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$$

$$\boxed{= \frac{3}{2}\ln|x^2+1| - 3\tan^{-1}(x) + \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C}$$

27.) $\int_0^{\pi/2} \cos^3 x \sin 2x\,dx = \int_0^{\pi/2} \cos^3(x)\,2\sin(x)\cos(x)\,dx = \int_0^{\pi/2} 2\cos^4(x)\sin(x)\,dx$

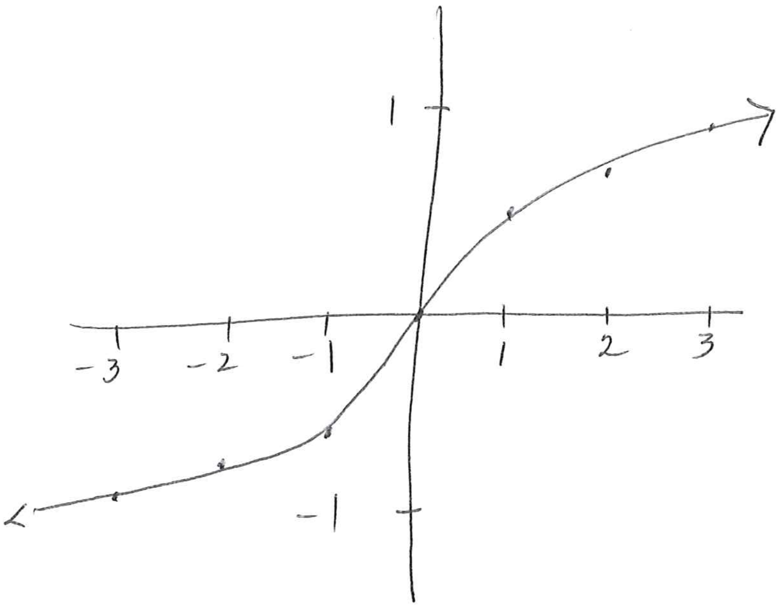
$u = \cos x$

$du = -\sin(x)\,dx$

$$-\int_1^0 2u^4\,du = 2\int_0^1 u^4\,du = \frac{2}{5}u^5\Big]_0^1 = \boxed{\frac{2}{5}}$$

29.) $\int_{-3}^{3} \frac{x}{1+|x|}\,dx$ This is an odd function $\therefore \int_{-3}^{3} \frac{x}{1+|x|}\,dx = 0$

x	$f(x)$
0	0
1	1/2
2	2/3
3	3/4
-1	-1/2
-2	-2/3
-3	-3/4



55.) $\int \sqrt{4x^2-4x-3}\,dx = \int \sqrt{(2x-1)^2-4}\,dx \quad \begin{pmatrix} u = 2x-1 \\ du = 2\,dx \\ \frac{1}{2}du = dx \end{pmatrix} =$

$\int \sqrt{u^2-2^2}\left(\frac{1}{2}du\right) = \frac{1}{2}\int \sqrt{u^2-2^2}\,du \overset{(\#39)}{=} \frac{1}{2}\left(\frac{u}{2}\sqrt{u^2-2^2} - \frac{2^2}{2}\ln\left|u+\sqrt{u^2+2^2}\right|\right) + C$

$= \frac{1}{4}u\sqrt{u^2-4} - \ln\left|u+\sqrt{u^2-4}\right| + C$

$= \boxed{\frac{1}{4}(2x-1)\sqrt{4x^2-4x-3} - \ln\left|2x-1+\sqrt{4x^2-4x-3}\right| + C}$

57.) $\int \cos(x)\sqrt{4+\sin^2 x}\,dx = \int \sqrt{4+u^2}\,du \overset{(\#21)}{=} \frac{u}{2}\sqrt{2^2+u^2} + \frac{2^2}{2}\ln\left(u+\sqrt{2^2+u^2}\right) + C$

$\begin{pmatrix} u = \sin(x) \\ du = \cos(x)\,dx \end{pmatrix}$

$= \boxed{\frac{1}{2}\sin(x)\sqrt{4+\sin^2 x} + 2\ln\left(\sin(x)+\sqrt{4+\sin^2 x}\right) + C}$